On the Limitation of Receiver-Functions Method: Beyond Conventional Assumptions & Advanced Inversion Techniques

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Some passive-source receiver-based methods

- Teleseismic travel times
- Teleseismic receiver functions (P and S)
- Body wave earthquake interferometry
- Ambient noise dispersion
- Interstation method surface wave dispersion
Basin structure and geometry from nuclear blasts waveforms and teleseismic travel times

Rodgers et al. PAGEOPH 2006; Tkalčić et al., BSSA 2008
The benefits of RFs + SWD

Earth Vs structure can be inverted using:
1. Receiver functions (RF)
2. Surface wave dispersion curves
3. RF + dispersion curves (jointly) or other datasets

Different approaches to modeling

- Forward modeling
- Linearized inversion
- Grid-search
- Non-linear inversion with optimization
- Multi-step approach
IRFFM (Interactive RF Forward Modeling)

Multi-step approach

Linearized Inversion

Tkalčić et al., JGR 2006
Chen et al., JGR 2010
Stipčević et al., GJI 2011
Tkalčić et al., GJI 2011; GJI 2012
Lithospheric structure of Saudi Arabia, China, Australia & Croatia from multi-step modeling of RFs and SWs

- SE China (RFs combined with tomography)
- SE Australia (RFs combined with ambient noise)
- Croatia and Adriatic Sea
Advantages and limitations of RFs

Advantages

• A way to invert for Vs structure under a single station
• Sensitive to gradients (discontinuities) in Vs velocity
• A needed complement to crustal tomography
• RF + SW dispersion curves (jointly) or other datasets

Limitations of conventional methods

• Information limited to a volume beneath a single station
• Insensitive to absolute velocity unless SW are added
• Simplifications/assumptions often cannot explain real Earth (1. lack of data, 2. anisotropy, 3. dipping layers, 4. non-uniqueness and noise in the data)
1. Exploiting seismic signal and noise in an aseismic environment to constrain crustal structure

Young et al., GJI 2012
2. Dipping Moho

Moho depth determined using $H - \kappa$ (above) and NA (right) method
From Stipčević, PhD Thesis, paper in preparation
Stations with similar results obtained using H-K and NA methods

Stipčević et al., in preparation
Stations for which there is a large difference between the H-K i NA results

Stipčević et al., in preparation
Moho dip determined using NA algorithm

Stipčević et al., in preparation
Starting model:

Two 20km thick layers in the crust Moho at 40 km with 20° dip & 270° strike

Synthetics are calculated using Fredrickson and Bostock method assuming an isotropic medium, and synthetic RFs are determined by deconvolution.

These synthetic RF data are then linearly stacked and inverted for Earth structure using NA method introduced in Exercise 6.
Dipping Moho - synthetic experiment

Starting model:

Two 20km thick layers in the crust Moho at 40 km with 20° dip & 270° strike

Synthetics are calculated using the Fredrickson and Bostock method assuming an isotropic medium and synthetic RFs are determined by deconvolution.
Dipping Moho - Synthetic Experiment

This is a result of the NA inversion when laterally homogeneous horizontal layers are assumed.

It is also assumed that the Moho is horizontal (not dipping).

(Exercise 10 in synopsis)
Dipping Moho - Synthetic Experiment

Now inverting for the Moho dip and orientation
3. A multi-step approach including polarization anisotropy

TKALČIĆ ET AL., JGR 2006
4. Non-uniqueness etc.

Different approaches to inverse problems

- Forward modeling
- Linearized inversion
- Grid-search
- Non-linear inversion with optimization
- Multi-step approach
- Non-linear inversion with the Bayesian framework
- Transdimensional Bayes framework...hierarchical

Bayes theorem:

\[ p(m \mid d_{obs}) \propto p(d_{obs} \mid m) p(m) \]
The importance of knowing the data noise in trans-dimensional formulation
Hierarchical Models

- Relationship between data noise and model complexity
- Treating data noise $\sigma$ as an unknown in the problem

Data noise is uncorrelated

Likelihood function

$$p(d \mid m) \propto \frac{1}{\sqrt{(2\pi\sigma^2)^N}} \exp \left[ -\frac{\|d - g(m)\|^2}{2\sigma^2} \right]$$

Data noise = measurement uncertainty + modeling uncertainty
Covariance Matrix of Noise in Data

Data noise is correlated

\[
\Phi(m) = [d - g(m)]^T C_D^{-1} [d - g(m)]
\]

\[
p(m \mid d) = \frac{1}{\sqrt{(2\pi)^N |C_D|}} \exp\left[ -\frac{\Phi(m)}{2} \right]
\]
How do we parameterize \( C_D \)?

\[
C_D = \sigma^2 \begin{bmatrix}
1 & r & r^2 & \cdots & r^{N-1} \\
r & 1 & r & \cdots & r^{N-2} \\
r^2 & r & 1 & \cdots & r^{N-3} \\
& \vdots & & \ddots & \vdots \\
r^N & r^{N-1} & r^{N-2} & \cdots & 1
\end{bmatrix}
\]

2 parameters:

- Magnitude of noise \( \sigma \)
- Correlation of noise \( r \)
Synthetic experiment

Magnitude and correlation of noise are treated as unknowns

Solution is a large ensemble of models distributed according to the target distribution

Bodin et al., JGR 2012
Synthetic experiment

Magnitude and correlation of noise are unknown

Different ways to look at the solution

Bodin et al., JGR 2012
Synthetic experiment

Magnitude and correlation of noise are unknown

Probability distribution at 31 km

Receiver Function

Vs (Km/s)

Depth (km)

Time (s)

Mean

Solution for maximum number of models

Noisy Receiver Function

Ensemble Solution

True Model

Average Solution

Maximum Solution

Mean Solution for maximum number of models
Synthetic experiment

Algorithm is able to recover the complexity of the model and the level of data noise.
Synthetic experiment

Magnitude and correlation of noise are unknown

Trade-offs between parameters
Application to field data

Bodin et al., JGR 2012
Application to Joint Inversion

Dispersion curve for Rayleigh waves

\[ d = d_{RF} + d_{dis} \]

\[ C_D = \begin{bmatrix} C_{RF} & 0 \\ 0 & C_{dis} \end{bmatrix} \]

Algorithm naturally weights the information brought by each data type.
Application to Joint Inversion (synthetic test)

Bodin et al., JGR 2012
Application to Joint Inversion

WOMBAT data from SE Australia: RFs + ambient noise dispersion

Bodin et al., JGR 2012
Future research

Modelling multiple geophysical datasets will be approached through the transdimensional hierarchical Bayesian framework, where the number of free parameters and the data noise will be treated as unknowns in the inversion.

Various simplifications that hinder the progress in crustal and lithospheric imaging using passive-source data and permanent/temporary seismic receivers will be gradually incorporated in the Bayesian inversion strategy - this includes, but is not limited to: anisotropy, dipping layers and 3D structure, noisy data, etc.

This is a general strategy that can be applied to other types of inverse problems in Earth Science and for imaging various parts of the Earth’s crust.